The Normal Distribution

Lecture 20
Section 6.3.1

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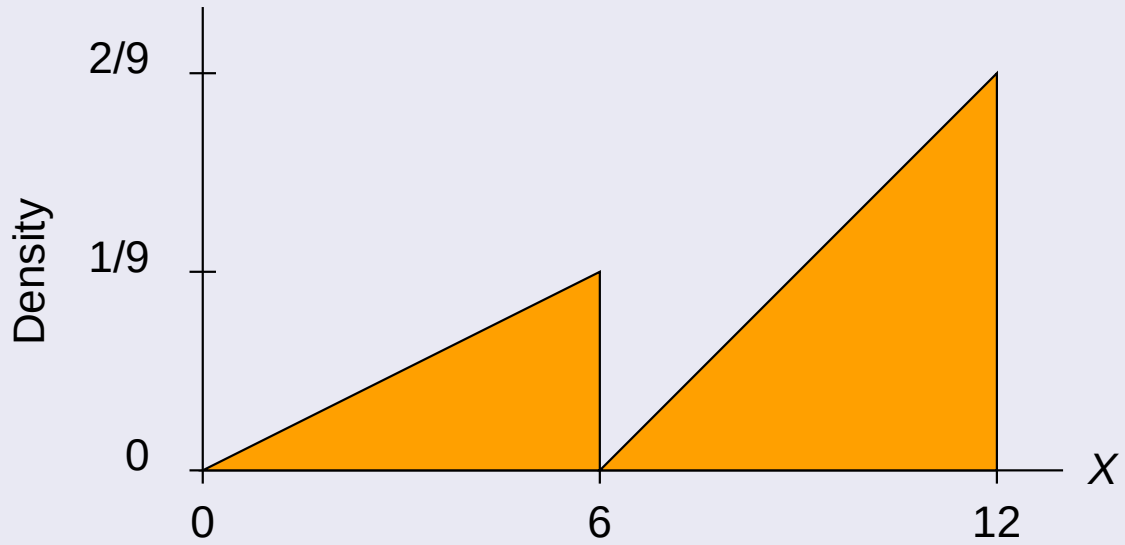
Wed, Oct 1, 2008

Outline

Homework Review

Exercise 6.3, page 376.

Consider the following density function for the variable X:



Homework Review

Exercise 6.3, page 376.

(a) The median is (select one)

(i) equal to 6.
(ii) less than 6.
(iii) more than 6.

(b) Explain your answer to part (a) using one well-written sentence.

(c) We are interested in learning about the proportion of units with values between 9 and 12. Explain in words or show on a graph what corresponds to this proportion; then calculate the proportion.

Homework Review

Solution

(a) The median is more than 6.

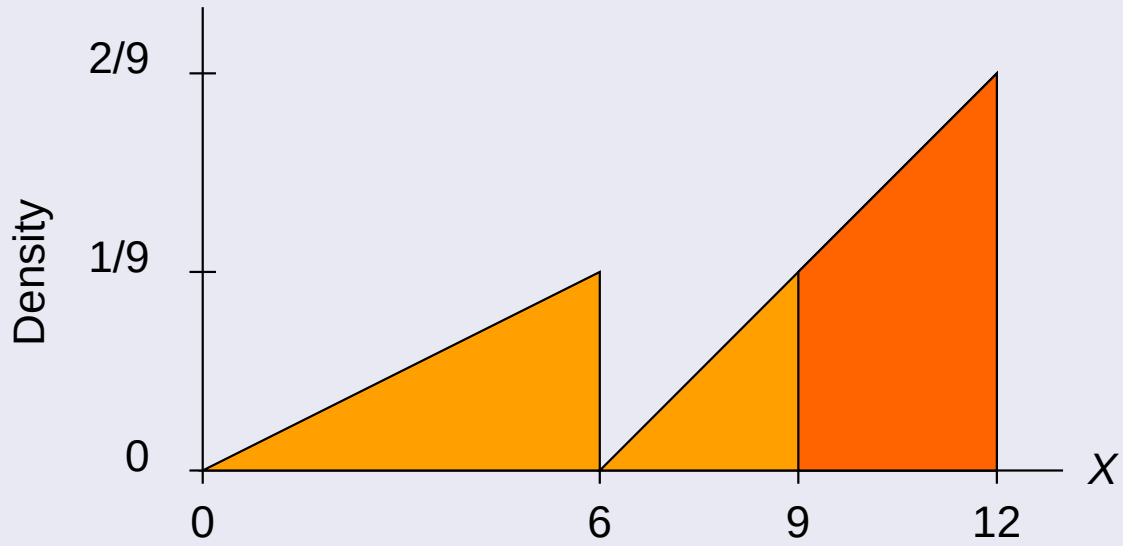
(b)
- It is clear from the graph that more than half the area is greater than 6, so the median must be greater than 6.
- In fact, 6 is the 33rd percentile. That is because the right-hand rectangle has twice the area of the left-hand rectangle.

Homework Review

Solution

(c)

- The area under the graph and between 9 and 12 corresponds to the proportion.



Homework Review

Solution

(c)

- Using the formula $A = \frac{1}{2}bh$ for the area of a triangle, the right-hand triangle has area

$$\frac{1}{2}(6)\left(\frac{2}{9}\right) = \frac{2}{3}.$$

- The triangle from 6 to 9 has area

$$\frac{1}{2}(3)\left(\frac{1}{9}\right) = \frac{1}{6}.$$

- Therefore, the area between 9 and 12 is

$$\frac{2}{3} - \frac{1}{6} = \frac{1}{2}.$$

The “68-95-99.7 Rule”

The “68-95-99.7 Rule”

For any normal distribution,

- Approximately 68% of the values lie within one standard deviation of the mean.
- Approximately 95% of the values lie within two standard deviations of the mean.
- Approximately 99.7% of the values lie within three standard deviations of the mean.

The Empirical Rule

- The well-known Empirical Rule is similar, but more general.

The Empirical Rule

For any "mound-shaped" distribution,

- Approximately 68% lie within one standard deviation of the mean.
- Approximately 95% lie within two standard deviations of the mean.
- Nearly all lie within three standard deviations of the mean.

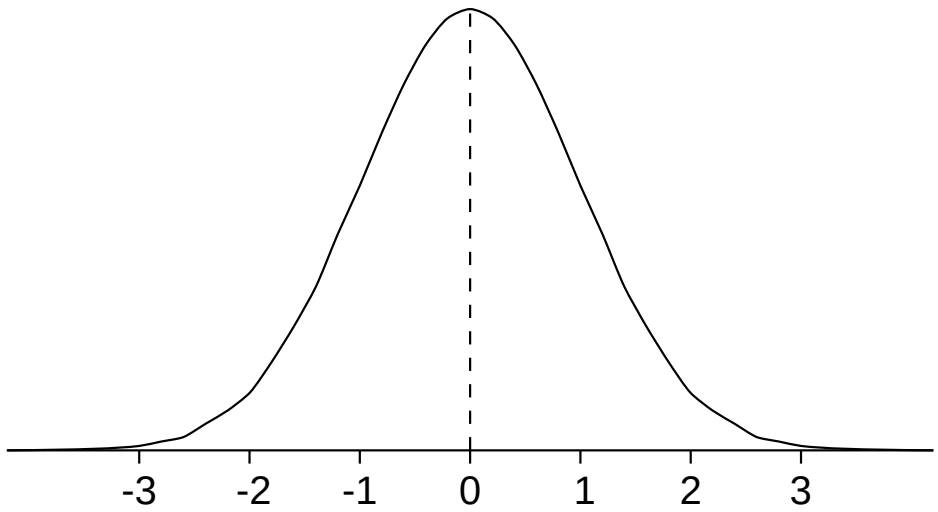
The Standard Normal Distribution

Definition (Standard normal distribution)

The standard normal distribution is the normal distribution with mean 0 and standard deviation 1.

- It is denoted by the letter Z.
- That is, Z is $N(0, 1)$.

The Standard Normal Distribution

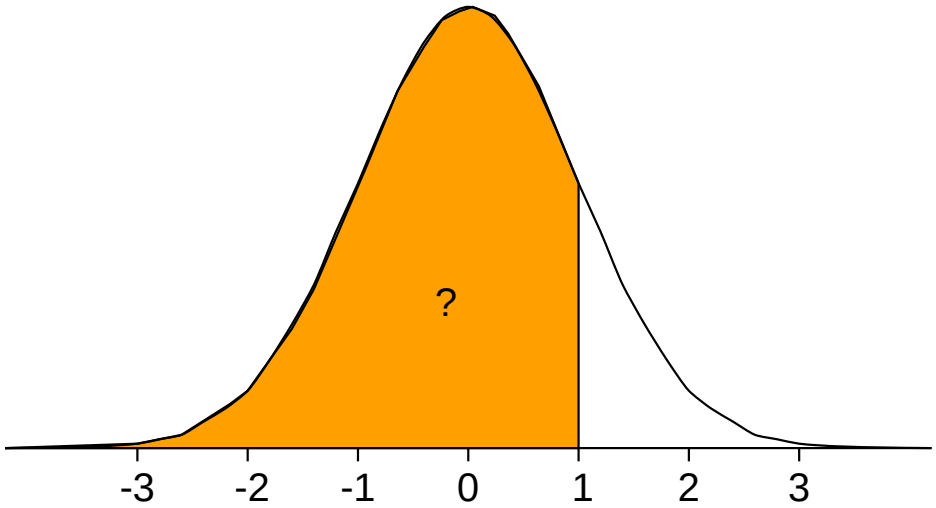


Areas Under the Standard Normal Curve

- Easy questions:
  - What is the total area under the curve?
  - What proportion of values of Z will fall below 0?
  - What proportion of values of Z will fall above 0?

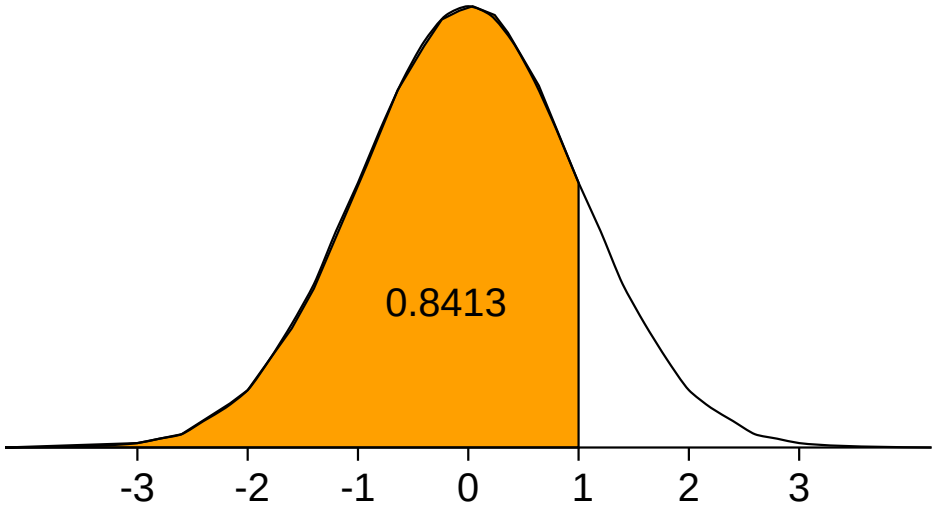
Areas Under the Standard Normal Curve

- What proportion of values will fall below $+1$?



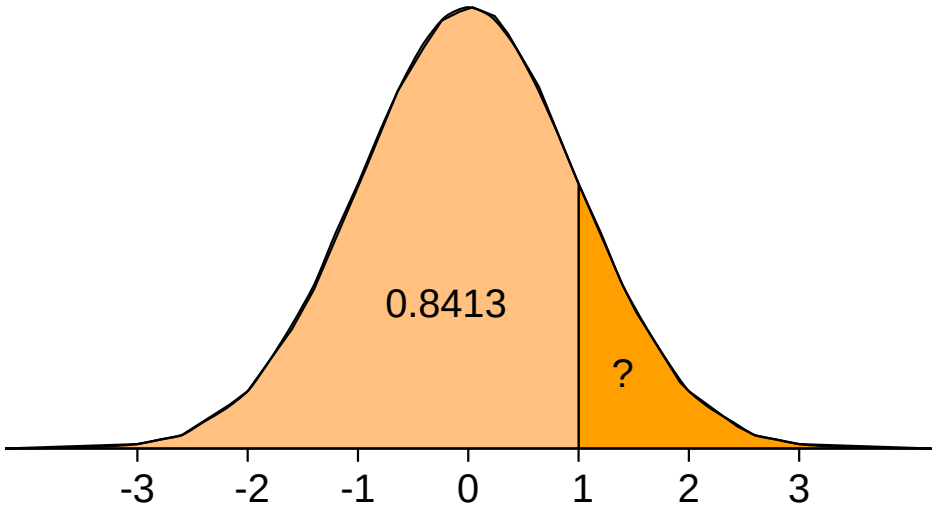
Areas Under the Standard Normal Curve

- It turns out that the area to the left of $+1$ is 0.8413.



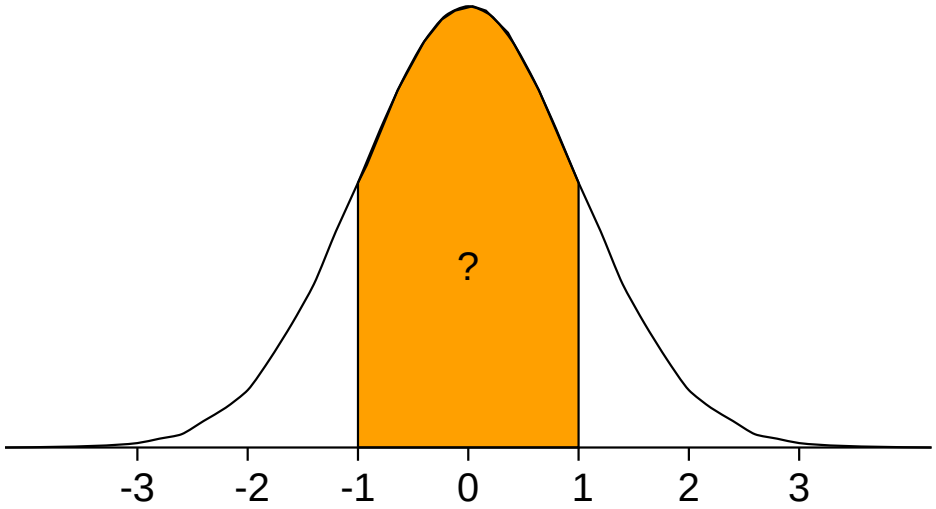
Areas Under the Standard Normal Curve

- What is the area to the right of $+1$?



Areas Under the Standard Normal Curve

- What is the area between -1 and 1?



Areas Under the Standard Normal Curve

- There are two methods to finding standard normal areas:
  - The TI-83 function `normalcdf`.
  - Standard normal table.
- We will use the TI-83 (unless you want to use the table).

TI-83 - Standard Normal Areas

TI-83 Standard Normal Areas

- Press `2nd DISTR`.
- Select `normalcdf` (Item #2).
- Enter the lower and upper bounds of the interval.
  - If the interval is infinite to the left, enter `−E99` as the lower bound.
  - If the interval is infinite to the right, enter `E99` as the upper bound.
- Press `ENTER`. The area appears in the display.

Standard Normal Areas

Practice

- Use the TI-83 to find the following.
  - The area between -1 and 1.
  - The area to the right of 1.
  - The area to the left of 1.645.
- What "standard normal" percentile is 1.645?

Other Normal Curves

- If we are working with a different normal distribution, say $N(30, 5)$, then how can we find areas under the curve?
- Use the same procedure as before, except enter the mean and standard deviation as the 3rd and 4th parameters of the `normalcdf` function.

TI-83 - Area Under Normal Curves

Example

TI-83 Normal Areas

- Find area between 25 and 38 in the distribution $N(30, 5)$.
- In the TI-83, enter `normalcdf(25,38,30,5)`.
- Press `ENTER`. The answer 0.7865 appears.

IQ Scores

- Read the article *Understanding and Interpreting IQ*.
- IQ scores are "normalized" to have a mean of 100 and a standard deviation of 15.
- Psychologists often assume a normal distribution of IQ scores as well.

IQ Scores

Practice

- What percentage of the population has an IQ above 120? above 140?
- What percentage of the population has an IQ between 75 and 125?

Assignment

Homework

- Read Section 6.3.1, pages 364 - 370.
- Let's Do It! 6.2, 6.3, 6.4, 6.5, 6.6
- Exercises 4 - 9, 11, 12, 15, 16, 18, page 376.